

1 FINANCE APPLICATIONS OF GAME THEORY

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Finance is concerned with how the savings of investors are allocated through financial markets and intermediaries to firms, which use them to fund their activities. Finance can be broadly divided into two fields. The first is *asset pricing*, which is concerned with the decisions of investors. The second is *corporate finance*, which is concerned with the decisions of firms. Traditional neoclassical economics did not attach much importance to either kind of finance. It was more concerned with the production, pricing and allocation of inputs and outputs and the operation of the markets for these. Models assumed certainty and in this context financial decisions are relatively straightforward. However, even with this simple methodology, important concepts such as the time value of money and discounting were developed.

Finance developed as a field in its own right with the introduction of uncertainty into asset pricing and the recognition that classical analysis failed to explain many aspects of corporate finance. In Section 1, we review the set of issues raised and some of the remaining problems with the pre-game theoretic literature. In Section 2, we recount how a first generation of game theory models tackled those problems, and discuss the successes and failures. Our purpose in this section is to point to some of the main themes in the various sub-fields. We do not attempt to provide an introduction to game theory. See Gibbons (1992) for a general introduction to applied game theory and Thakor (1991) for a survey of game theory in finance including an introduction to game theory. Nor do we attempt to be encyclopedic.

This first generation of game theoretic models revolutionized finance but much remains to be explained. Game theoretic methods continue to develop and we believe that extensions involving richer informational models are especially relevant for finance. In Section 3, we review recent work concerning higher order beliefs and informational cascades and discuss its relevance for finance. We also review work that entails differences in beliefs not explained by differences in information.

1. The Main Issues in Finance

Asset Pricing

The focus of Keynesian macroeconomics on uncertainty and the operation of financial markets led to the development of frameworks for analyzing risk. Keynes (1936) and Hicks (1939) took account of risk by adding a risk premium to the interest rate. However, there was no systematic theory underlying this risk premium. The key theoretical development which eventually led to such a theory was von Neumann and Morgenstern's (1947) axiomatic approach to choice under uncertainty. Their notion of expected utility, developed originally for use in game theory, underlies the vast majority of theories of asset pricing.

The Capital Asset Pricing Model. Markowitz (1952; 1959) utilized a special case of von Neumann and Morgenstern's expected utility to develop a theory of portfolio choice. He considered the case where investors are only concerned with the mean and variance of the payoffs of the portfolios they are choosing. This is a special case of expected utility provided the investor's utility of consumption is quadratic and/or asset returns are multinormally distributed. Markowitz's main result was to show that diversifying holdings is optimal and the benefit that can be obtained depends on the covariances of asset returns. Tobin's (1958) work on liquidity preference helped to establish the mean-variance framework as the standard approach to portfolio choice problems. Subsequent authors have made extensive contributions to portfolio theory. See Constantinides and Malliaris (1995).

It was not until some time after Markowitz's original contribution that his framework of individual portfolio choice was used as the basis for an equilibrium theory, namely the capital asset pricing model (CAPM). Brennan

(1989) has argued that the reason for the delay was the boldness of the assumption that all investors have the same beliefs about the means and variances of all assets. Sharpe (1964) and Lintner (1965) showed that in equilibrium

$$Er_i = r_f + \beta_i(Er_M - r_f),$$

where Er_i is the expected return on asset i , r_f is the return on the risk free asset, Er_M is the expected return on the market portfolio (i.e. a value weighted portfolio of all assets in the market) and $\beta_i = \text{cov}(r_i, r_M)/\text{var}(r_M)$. Black (1972) demonstrated that the same relationship held even if no risk free asset existed provided r_f was replaced by the expected return on a portfolio or asset with $\beta = 0$. The model formalizes the risk premium of Keynes and Hicks and shows that it depends on the covariance of returns with other assets.

Despite being based on the very strong assumptions of mean-variance preferences and homogeneity of investor beliefs, the CAPM was an extremely important development in finance. It not only provided key theoretical insights concerning the pricing of stocks but also led to a great deal of empirical work testing whether these predictions held in practice. Early tests such as Fama and Macbeth (1973) provided some support for the model. Subsequent tests using more sophisticated econometric techniques have not been so encouraging. Ferson (1995) contains a review of these tests.

The CAPM is only one of many asset-pricing models that have been developed. Other models include the Arbitrage Pricing Theory (APT) of Ross (1977a) and the representative agent asset-pricing model of Lucas (1978). However, the CAPM was the most important not only because it was useful in its own right for such things as deriving discount rates for capital budgeting but also because it allowed investigators to easily adjust for risk when considering a variety of topics. We turn next to one of the most important hypotheses that resulted from this ability to adjust for risk.

Market Efficiency. In models with competitive markets, symmetric information and no frictions such as transaction costs, the only variations in returns across assets are due to differences in risk. All information that is available to investors becomes reflected in stock prices and no investor can earn higher returns except by bearing more risk. In the CAPM, for example, it is only differences in β 's that cause differences in returns. The idea that the differences in returns are due to differences in risk came to be known as the *Efficient Markets Hypothesis*. During the 1960's a considerable amount of research was undertaken to see whether U.S. stock markets were in fact

efficient. In a well-known survey, Fama (1970) argued that the balance of the evidence suggested markets were efficient. In a follow up piece, Fama (1991) continued to argue that by and large markets were efficient despite the continued documentation of numerous anomalies.

Standard tests of market efficiency involve a joint test of market efficiency and the equilibrium asset-pricing model that is used in the analysis. Hence a rejection of the joint hypothesis can either be a rejection of market efficiency or the asset-pricing model used or both. Hawawini and Keim (1995) survey these “anomalies.” Basu (1977) discovered one of the first. He pointed out that price to earnings (P/E) ratios provided more explanatory power than β 's. Firms with low P/E ratios (value stocks) tend to outperform stocks with high P/E ratios (growth stocks). Banz (1981) showed that there was a significant relationship between the market value of common equity and returns (the size effect). Stattman (1980) and others have demonstrated the significant predictive ability of price per share to book value per share (P/B) ratios for returns. In an influential paper, Fama and French (1993) have documented that firm size and the ratio of book to market equity are important factors in explaining average stock returns. In addition to these cross-sectional effects there are also a number of significant time series anomalies. Perhaps the best known of these is the January effect. Rozeff and Kinney (1976) found that returns on an equal weighted index of NYSE stocks were much higher in January than in the other months of the year. Keim (1983) demonstrated that the size effect was concentrated in January. Cross (1973) and French (1980) pointed out that the returns on the S&P composite index are negative on Mondays. Numerous other studies have confirmed this weekend effect in a wide variety of circumstances.

These anomalies are difficult to reconcile with models of asset pricing such as the CAPM. Most of them are poorly understood. Attempts have been made to explain the January effect by tax loss selling at the end of the year. Even this is problematic because in countries such as the U.K. and Australia where the tax year does not end in December there is still a January effect. It would seem that the simple frameworks most asset pricing models adopt are not sufficient to capture the richness of the processes underlying stock price formation.

Instead of trying to reconcile these anomalies with asset pricing theories based on rational behavior, a number of authors have sought to explain them using behavioral theories based on foundations taken from the psychology

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literature. For example, Dreman (1982) argues that the P/E effect can be explained by investors' tendency to make extreme forecasts. High (low) P/E ratio stocks correspond to a forecast of high (low) growth by the market. If investors predict too high (low) growth, high P/E stocks will underperform (overperform). De Bondt and Thaler (1995) surveys behavioral explanations for this and other anomalies.

Continuous Time Models. Perhaps the most significant advance in asset pricing theory since the early models were formulated was the extension of the paradigm to allow for continuous trading. This approach was developed in a series of papers by Merton (1969; 1971; 1973a) and culminated in his development of the intertemporal capital asset pricing model (ICAPM). The assumptions of expected utility maximization, symmetric information and frictionless markets are maintained. By analyzing both the consumption and portfolio decisions of an investor through time and assuming prices per share are generated by Ito processes, greater realism and tractability compared to the mean-variance approach is achieved. In particular, it is not necessary to assume quadratic utility or normally distributed returns. Other important contributions that were developed using this framework were Breeden's (1979) Consumption CAPM and Cox, Ingersoll and Ross's (1985) modeling of the term structure of interest rates.

The relationship between continuous time models and the Arrow–Debreu general equilibrium model was considered by Harrison and Kreps (1979) and Duffie and Huang (1985). Repeated trading allows markets to be made effectively complete even though there are only a few securities.

One of the most important uses of continuous time techniques is for the pricing of derivative securities such as options. This was pioneered by Merton (1973b) and Black and Scholes (1973) and led to the development of a large literature that is surveyed in Ross (1992). Not only has this work provided great theoretical insight but it has also proved to be empirically implementable and of great practical use.

Corporate Finance

The second important area considered by finance is concerned with the financial decisions made by firms. These include the choice between debt and equity and the amount to pay out in dividends. The seminal work in this area was Modigliani and Miller (1958) and Miller and Modigliani (1961). They

showed that with perfect markets (i.e., no frictions and symmetric information) and no taxes the total value of a firm is independent of its debt/equity ratio. Similarly they demonstrated that the value of the firm is independent of the level of dividends. In their framework it is the investment decisions of the firm that are important in determining its total value.

The importance of the Modigliani and Miller theorems was not as a description of reality. Instead it was to stress the importance of taxes and capital market imperfections in determining corporate financial policies. Incorporating the tax deductibility of interest but not dividends and bankruptcy costs lead to the trade-off theory of capital structure. Some debt is desirable because of the tax shield arising from interest deductibility but the costs of bankruptcy and financial distress limit the amount that should be used. With regard to dividend policy, incorporating the fact that capital gains are taxed less at the personal level than dividends into the Modigliani and Miller framework gives the result that all payouts should be made by repurchasing shares rather than by paying dividends.

The trade-off theory of capital structure does not provide a satisfactory explanation of what firms do in practice. The tax advantage of debt relative to the magnitude of expected bankruptcy costs would seem to be such that firms should use more debt than is actually observed. Attempts to explain this, such as M. Miller (1977), that incorporate personal as well as corporate taxes into the theory of capital structure, have not been successful. In the Miller model, there is a personal tax advantage to equity because capital gains are only taxed on realization and a corporate tax advantage to debt because interest is tax deductible. In equilibrium, people with personal tax rates above the corporate tax rate hold equity while those with rates below hold debt. This prediction is not consistent with what occurred in the U.S. in the late 1980's and early 1990's when there were no personal tax rates above the corporate rate. The Miller model suggests that there should have been a very large increase in the amount of debt used by corporations but there was only a small change.

The tax-augmented theory of dividends also does not provide a good explanation of what actually happens. Firms have paid out a substantial amount of their earnings as dividends for many decades. Attempts to explain the puzzle using tax based theories such as the clientele model have not been found convincing. They are difficult to reconcile with the fact that many

people in high tax brackets hold large amounts of dividend paying stocks and on the margin pay significant taxes on the dividends.

Within the Modigliani and Miller framework other corporate financial decisions also do not create value except through tax effects and reductions in frictions such as transaction costs. Although theoretical insights are provided, the theories are not consistent with what is observed in practice. As with the asset pricing models discussed above this is perhaps not surprising given their simplicity. In particular, the assumptions of perfect information and perfect markets are very strong.

2. The Game-Theory Approach

The inability of standard finance theories to provide satisfactory explanations for observed phenomena lead to a search for theories using new methodologies. This was particularly true in corporate finance where the existing models were so clearly unsatisfactory. Game theory has provided a methodology that has brought insights into many previously unexplained phenomena by allowing asymmetric information and strategic interaction to be incorporated into the analysis. We start with a discussion of the use of game theory in corporate finance where to date it has been most successfully applied. We subsequently consider its role in asset pricing.

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Dividends as Signals. The thorniest issue in finance has been what Black (1976) termed “the dividend puzzle.” Firms have historically paid out about a half of their earnings as dividends. Many of these dividends were received by investors in high tax brackets who, on the margin, paid substantial amounts of taxes on them. In addition, in a classic study Lintner (1956) demonstrated that managers “smooth” dividends in the sense that they are less variable than earnings. This finding was confirmed by Fama and Blasiak (1968) and numerous other authors. The puzzle has been to explain these observations. See Allen and Michaely (1995) for a survey of this literature.

In their original article on dividends, Miller and Modigliani (1961) had suggested that dividends might convey significant information about a firm’s prospects. However, it was not until game theoretic methods were applied that any progress was made in understanding this issue. Bhattacharya’s

(1979) model of dividends as a signal was one of the first papers in finance to use these tools. His contribution started a large literature.

Bhattacharya assumes that managers have superior information about the profitability of their firm's investment. They can signal this to the capital market by "committing" to a sufficiently high level of dividends. If it turns out the project is profitable these dividends can be paid from earnings without a problem. If the project is unprofitable then the firm has to resort to outside finance and incur deadweight transaction costs. The firm will therefore only find it worthwhile to commit to a high dividend level if in fact its prospects are good. Subsequent authors like Miller and Rock (1985) and John and Williams (1985) developed models which did not require committing to a certain level of dividends and where the deadweight costs required to make the signal credible were plausible.

One of the problems with signaling models of dividends is that they typically suggest that dividends will be paid to signal new information. Unless new information is continually arriving there is no need to keep paying them. But in that case the level of dividends should be varying to reflect the new information. This feature of dividend signaling models is difficult to reconcile with smoothing. In an important piece, Kumar (1988) develops a 'coarse signaling' theory that is consistent with the fact that firms smooth dividends. Firms within a range of productivity all pay the same level of dividends. It is only when they move outside this range that they will alter their dividend level.

Another problem in many dividend signaling models (including Kumar (1988)) is that they do not explain why firms use dividends rather than share repurchases. In most models the two are essentially equivalent except for the way that they are taxed since both involve transferring cash from the firm to the owners. Dividends are typically treated as ordinary income and taxed at high rates whereas repurchases involve price appreciations being taxed at low capital gains rates. Building on work by Ofer and Thakor (1987) and Barclay and Smith (1988), Brennan and Thakor (1990) suggest that repurchases have a disadvantage in that informed investors are able to bid for undervalued stocks and avoid overvalued ones. There is thus an adverse selection problem. Dividends do not suffer from this problem because they are pro rata.

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Some progress in understanding the dividend puzzle has been made in recent years. This is one of the finance applications of game theory that has been somewhat successful.

Capital Structure. The trade-off theory of capital structure mentioned above has been a textbook staple for many years. Even though it had provided a better explanation of firms' choices than the initial dividend models, the theory is not entirely satisfactory because the empirical magnitudes of bankruptcy costs and interest tax shields do not seem to match observed capital structures. The use of game theoretic techniques in this field has allowed it to move ahead significantly. Harris and Raviv (1991) survey the area.

The first contributions in a game-theoretic vein were signaling models. Ross (1977b) develops a model where managers signal the prospects of the firm to the capital markets by choosing an appropriate level of debt. The reason this acts as a signal is that bankruptcy is costly. A high debt firm with good prospects will only incur these costs occasionally while a similarly levered firm with poor prospects will incur them often. Leland and Pyle (1977) consider a situation where entrepreneurs use their retained share of ownership in a firm to signal its value. Owners of high-value firms retain a high share of the firm to signal their type. Their high retention means they don't get to diversify as much as they would if there was symmetric information, and it is this that makes it unattractive for low value firms to mimic them.

Two influential papers based on asymmetric information are Myers (1984) and Myers and Majluf (1984). If managers are better informed about the prospects of the firm than the capital markets, they will be unwilling to issue equity to finance investment projects if the equity is undervalued. Instead they will have a preference for using equity when it is overvalued. Thus equity is regarded as a bad signal. Myers (1984) uses this kind of reasoning to develop the "pecking order" theory of financing. Instead of using equity to finance investment projects, it will be better to use less information sensitive sources of funds. Retained earnings are the most preferred, with debt coming next and finally equity. The results of these papers and the subsequent literature such as Stein (1992) and Nyborg (1995) are consistent with a number of stylized facts concerning the effect of issuing different types of security on stock price and the financing choices of firms. However, in order to derive them, strong assumptions such as overwhelming bankruptcy aversion of managers are often necessary. Moreover, as Dybvig and Zender

(1991) and others have stressed, they often assume sub-optimal managerial incentive schemes. Dybvig and Zender show that if managerial incentive schemes are chosen optimally, the Modigliani and Miller irrelevance results can hold even with asymmetric information.

A second contribution of game theory to understanding capital structure lies in the study of agency costs. Jensen and Meckling (1976) pointed to two kinds of agency problems in corporations. One is between equity holders and bondholders and the other is between equity holders and managers. The first arises because the owners of a levered firm have an incentive to take risks; they receive the surplus when returns are high but the bondholders bear the cost when default occurs. Diamond (1989) has shown how reputation considerations can ameliorate this risk shifting incentive when there is a long time horizon. The second conflict arises when equity holders cannot fully control the actions of managers. This means that managers have an incentive to pursue their own interests rather than those of the equity holders. Grossman and Hart (1982) and Jensen (1986) among others have shown how debt can be used to help overcome this problem. Myers (1977) has pointed to a third agency problem. If there is a large amount of debt outstanding which is not backed by cash flows from the firm's assets, i.e. a "debt overhang," equity holders may be reluctant to take on safe, profitable projects because the bondholders will have claim to a large part of the cash flows from these.

The agency perspective has also led to a series of important papers by Hart and Moore and others on financial contracts. These use game-theoretic techniques to shed light on the role of incomplete contracting possibilities in determining financial contracts and in particular debt. Hart and Moore (1989) consider an entrepreneur who wishes to raise funds to undertake a project. Both the entrepreneur and the outside investor can observe the project payoffs at each date, but they cannot write explicit contracts based on these payoffs because third parties such as courts cannot observe them. The focus of their analysis is the problem of providing an incentive for the entrepreneur to repay the borrowed funds. Among other things, it is shown that the optimal contract is a debt contract and incentives to repay are provided by the ability of the creditor to seize the entrepreneur's assets. Subsequent contributions include Hart and Moore (1994; 1998), Aghion and Bolton (1992), Berglof and von Thadden (1994) and von Thadden (1995). Hart (1995) contains an excellent account of the main ideas in this literature.

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The Modigliani and Miller (1958) theory of capital structure is such that the product market decisions of firms are separated from financial market decisions. Essentially this is achieved by assuming there is perfect competition in product markets. In an oligopolistic industry where there are strategic interactions between firms in the product market, financial decisions are also likely to play an important role. Allen (1986), Brander and Lewis (1986) and Maksimovic (1986) and a growing subsequent literature (see Maksimovic (1995) for a survey) have considered different aspects of these interactions between financing and product markets. Allen (1986) considers a duopoly model where a bankrupt firm is at a strategic disadvantage in choosing its investment because the bankruptcy process forces it to delay its decision. The bankrupt firm becomes a follower in a Stackelberg investment game instead of a simultaneous mover in a Nash-Cournot game. Brander and Lewis (1986) and Maksimovic (1986) analyze the role of debt as a precommitment device in oligopoly models. By taking on a large amount of debt a firm effectively precommits to a higher level of output. Titman (1984) and Maksimovic and Titman (1993) have considered the interaction between financial decisions and customers' decisions. Titman (1984) looks at the effect of an increased probability of bankruptcy on product price because, for example, of the difficulties of obtaining spare parts and service should the firm cease to exist. Maksimovic and Titman (1993) consider the relationship between capital structure and a firm's reputational incentives to maintain high product quality.

A significant component of the trade-off theory is the bankruptcy costs that limit the use of debt. An important issue concerns the nature of these bankruptcy costs. Haugen and Senbet (1978) argued that the extent of bankruptcy costs was limited because firms could simply renegotiate the terms of the debt and avoid bankruptcy and its associated costs. The literature on strategic behavior around and within bankruptcy relies extensively on game-theoretic techniques. See Webb (1987), Giammarino (1988), Brown (1989) and, for a survey, Senbet and Seward (1995). This work shows that Haugen and Senbet's argument depends on the absence of frictions. With asymmetric information or other frictions, bankruptcy costs can occur in equilibrium.

The Market for Corporate Control. The concept of the market for corporate control was first developed by Manne (1965). He argued that in order for resources to be used efficiently, it is necessary that firms be run by the most able and competent managers. Manne suggests that the way in which modern

capitalist economies achieve this is through the market for corporate control. There are several ways in which this operates including tender offers, mergers and proxy fights.

Traditional finance theory with its assumptions of symmetric information and perfectly competitive frictionless capital markets had very little to offer in terms of insights into the market for corporate control. In fact the large premiums over initial stock market valuations paid for targets appeared to be at variance with market efficiency and posed something of a puzzle. Again it was not until the advent of game theoretic concepts and techniques that much progress was made in this area.

The paper that provided a formal model of the takeover process and renewed interest in the area was Grossman and Hart (1980). They pointed out that the tender offer mechanism involved a free rider problem. If a firm makes a bid for a target in order to replace its management and run it more efficiently then each of the target's shareholders has an incentive to hold out and say no to the bid. The reason is that they will then be able to benefit from the improvements implemented by the new management. They will only be willing to tender if the offer price fully reflects the value under the new management. Hence a bidding firm cannot make a profit from tendering for the target. In fact if there are costs of acquiring information in preparation for the bid or other bidding costs, the firm will make a loss. The free rider problem thus appears to exclude the possibility of takeovers. Grossman and Hart's solution to this dilemma was that a firm's corporate charter should allow acquirors to obtain benefits unavailable to other shareholders after the acquisition. They term this process "dilution."

Another solution to the free rider problem, pointed out by Shleifer and Vishny (1986a), is for bidders to be shareholders in the target before making any formal tender offer. In this way they can benefit from the price appreciation in the "toehold" of shares they already own even if they pay full price for the remaining shares they need to acquire. The empirical evidence is not consistent with this argument, however. Bradley, Desai and Kim (1988) find that the majority of bidders own no shares prior to the tender offer.

A second puzzle that the empirical literature has documented is the fact that bidding in takeover contests occurs through several large jumps rather than many small ones. For example, Jennings and Mazzeo (1993) found that the majority of the initial bid premiums exceed 20% of the market value of the

target 10 days before the offer. This evidence conflicts with the standard solution of the English auction model that suggests there should be many small bid increments. Fishman (1988) argues that the reason for the large initial premium is to deter potential competitors. In his model, observing a bid alerts the market to the potential desirability of the target. If the initial bid is low a second bidder will find it worthwhile to spend the cost to investigate the target. This second firm may then bid for the target and push out the first bidder or force a higher price to be paid. By starting with a sufficiently high bid the initial bidder can reduce the likelihood of this competition.

Much of the theoretical literature has attempted to explain why the defensive measures that many targets adopt may be optimal for their shareholders. Typically the defensive measures are designed to ensure that the bidder that values the company the most ends up buying it. For example, Shleifer and Vishny (1986b) develop a model where the payment of greenmail to a bidder, signals to other interested parties that no “white knight” is waiting to buy the firm. This puts the firm in play and can lead to a higher price being paid for it than initially would have been the case.

A survey of the literature on takeovers is contained in Hirshleifer (1995). Since strategic interaction and asymmetric information are the essence of takeover contests, game theory has been central to the literature.

Initial Public Offerings (IPOs). In 1963 the U.S. Securities and Exchange Commission undertook a study of IPOs and found that the initial short-run return on these stocks was significantly positive. Logue (1973), Ibbotson (1975) and numerous subsequent academic studies have found a similar result. In a survey of the literature on IPOs, Ibbotson and Ritter (1995) give a figure of 15.3% for the average increase in the stock price during the first day of trading based on data from 1960-1992. The large short run return on IPOs was for many years one of the most glaring challenges to market efficiency. The standard symmetric information models that existed in the 1960s and 1970s were not at all consistent with this observation.

The first paper to provide an appealing explanation of this phenomenon was Rock (1986). In his model the under-pricing occurs because of adverse selection. There are two groups of buyers for the shares, one is informed about the true value of the stock while the other is uninformed. The informed group will only buy when the offering price is at or below the true value.

This implies that the uninformed will receive a high allocation of overpriced stocks since they will be the only people in the market when the offering price is above the true value. Rock suggested that in order to induce the uninformed to participate they must be compensated for the overpriced stock they ended up buying. Under-pricing on average is one way of doing this.

Many other theories of under-pricing followed. These include under-pricing as a signal (Allen and Faulhaber (1989); Grinblatt and Hwang (1989) and Welch (1989)), as an inducement for investors to truthfully reveal their valuations (Benveniste and Spindt (1989)), to deter lawsuits (Hughes and Thakor (1992)), and to stabilize prices (Ruud (1993)), among others.

In addition to the short run under-pricing puzzle, there is another anomaly associated with IPOs. Ritter (1991) documents significant long-run under-performance of newly issued stocks. During 1975-1984, he finds a cumulative average under-performance of around 15% from the offer price relative to the matching firm-adjusted return. Loughran (1993) and Loughran and Ritter (1995) confirmed this long run under-performance in subsequent studies.

Several behavioral theories have also been put forward to explain long-run under-performance. E. Miller (1977) argues that there is a wide range of opinion concerning IPOs and the initial price will reflect the most optimistic opinion. As information is revealed through time, the most optimistic investors will gradually adjust their beliefs and the price of the stock will fall. Shiller (1990) argues that the market for IPOs is subject to an 'impresario' effect. Investment banks will try to create the appearance of excess demand and this will lead to a high price initially but subsequently to underperformance. Finally, Ritter (1991) and Loughran and Ritter (1995) suggest that there are swings of investor sentiment in the IPO market and firms use the "window of opportunity" created by overpricing to issue equity.

Although IPOs represent a relatively small part of financing activity, they have received a great deal of attention in the academic literature. The reason perhaps is the extent to which underpricing and overpricing represent a violation of market efficiency. It is interesting to note that while game theoretic techniques have provided many explanations of underpricing they have not been utilized to explain overpricing. Instead the explanations presented have relied on relaxing the assumption of rational behavior by investors.

Intermediation. A second area that has been significantly changed by game-theoretic models is intermediation. Traditionally, banks and other intermediaries were regarded as vehicles for reducing transaction costs (Gurley and Shaw (1960)). The initial descriptions of bank behavior were relatively limited. Indeed, the field was dramatically changed by the modeling techniques introduced in Diamond and Dybvig (1983). This paper develops a simple model where a bank provides insurance to depositors against liquidity shocks. At an intermediate date customers find out whether they require liquidity then or at the final date. There is a cost to liquidating long term assets at the intermediate date. A deposit contract is used where customers who withdraw first get the promised amount until resources are exhausted after which nothing is received (i.e., the first come first served constraint). These assumptions result in two self-fulfilling equilibria. In the good equilibrium, everybody believes only those who have liquidity needs at the intermediate date will withdraw their funds and this outcome is optimal for both types of depositor. In the bad equilibrium, everybody believes everybody else will withdraw. Given the assumptions of first come first served and costly liquidating of long-term assets, it is optimal for early and late consumers to withdraw and there is a run on the bank. Diamond and Dybvig argue the bad equilibrium can be eliminated by deposit insurance. In addition to being important as a theory of runs, the paper was also instrumental in modeling liquidity needs. Similar approaches have been adopted in the investigation of many topics.

Diamond and Dybvig (1983) together with an earlier paper Bryant (1980) led to a large literature on bank runs and panics. For example, Chari and Jagannathan (1988) consider the role of aggregate risk in causing bank runs. They focus on a signal extraction problem where part of the population observes a signal about the future returns of bank assets. Others must then try to deduce from observed withdrawals whether an unfavorable signal was received by this group or whether liquidity needs happen to be high. The authors are able to show that panics occur not only when the economic outlook is poor but also when liquidity needs turn out to be high. Jacklin and Bhattacharya (1988) compare what happens with bank deposits to what happens when securities are held directly so runs are not possible. In their model some depositors receive a signal about the risky investment. They show that either bank deposits or directly held securities can be optimal depending on the characteristics of the risky investment. The comparison of bank-based and stock market-based financial systems has become a widely

considered topic in recent years (see Thakor (1996) and Allen and Gale (1999)).

Other important papers in the banking and intermediation literature are Stiglitz and Weiss (1981) and Diamond (1984). The former paper developed an adverse selection model in which rationing credit is optimal. The latter paper considers a model of delegated monitoring where banks have an incentive to monitor borrowers because otherwise they will be unable to pay off depositors. A full account of the recent literature on banking is contained in Bhattacharya and Thakor (1993).

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Early work incorporating asymmetric information into the asset pricing literature employed the (non-strategic) concept of rational expectations equilibrium (Grossman and Stiglitz (1980)). Each market participant is assumed to learn from market prices but still believes that he does not influence market prices. This literature helped address a number of novel issues, for example, free riding in the acquisition of information. But a number of conceptual problems arose in attempting to reconcile asymmetric information with competitive analysis, and an explicitly strategic analysis seemed to be called for (Dubey, Geanakoplos and Shubik (1987)).

This provided one motive for the recent literature on market microstructure. Whereas general equilibrium theory simply assumes an abstract price formation mechanism, the market microstructure literature seeks to model the process of price formation in financial markets under explicit trading rules. The papers that contained the initial important contributions are Kyle (1985) and Glosten and Milgrom (1985). O'Hara (1995) provides an excellent survey of the extensive literature that builds on these two papers.

Kyle (1985) develops a model with a single risk neutral market maker, a group of noise traders who buy or sell for exogenous reasons such as liquidity needs and a risk-neutral informed trader. The market maker selects efficient prices, and the noise traders simply submit orders. The informed trader chooses a quantity to maximize his expected profit. In Glosten and Milgrom (1985) there are also a risk neutral market maker, noise traders, and informed traders. In contrast to Kyle's model, Glosten and Milgrom treat trading quantities as fixed and instead focus on the setting of bid and ask prices. The market maker sets the bid ask spread to take into account the

possibility that the trader may be informed and have a better estimate of the true value of the security. As orders are received, the bid and ask prices change to reflect the trader's informational advantage. In addition, the model is competitive in the sense that the market maker is constrained to make zero expected profits.

Besides the field of market microstructure, a number of other asset-pricing topics have been influenced by game theory. These include market manipulation models. See Cherian and Jarrow (1995) for a survey. Many financial innovation models, described for instance in Allen and Gale (1994) and Duffie and Rahi (1995), also use game theoretic techniques. However, these areas do not as yet have the visibility of other areas in asset pricing.

Pricing anomalies such as those associated with P/E or P/B ratios that have received so much attention in recent years are intimately associated with accounting numbers. Since these numbers are to some extent the outcome of strategic decisions, analysis of these phenomena using game-theoretic techniques seems likely to be a fruitful area of research.

3. Richer Models of Information and Beliefs

Despite the great progress in finance using game-theoretic techniques, many phenomena remain unexplained. One reaction to this has been to move away from models based on rational behavior and develop behavioral models. We argue that it is premature to abandon rationality. Recent developments in game theory have provided powerful new techniques that explain many important financial phenomena. In this section, we review three lines of research and consider their implications for finance.

Higher Order Beliefs

Conventional wisdom in financial markets holds that participants are concerned not just about fundamentals, but also about what others believe about fundamentals, what others believe about others' beliefs, and so on. Remarkably, the mainstream finance literature largely ignores such issues. When such concerns are introduced and discussed, it is usually in the context of models with irrational actors. Yet the game theory literature tells us that when there are co-ordination aspects to a strategic situation, such higher order beliefs are crucially important for fully rational actors.

How do these issues come to be bypassed? In our view, this happens because models of asymmetric information to date – though tractable and successful in examining many finance questions -- are not rich enough to address issues of higher order beliefs. If it is assumed that players' types, or signals, are independent, it is (implicitly) assumed that there is common knowledge of players' beliefs about other players' beliefs. If it is assumed that each signal that a player observes implies a different belief about fundamentals, it is (implicitly) assumed that a player's belief about others' beliefs is uniquely determined by his belief about fundamentals. Modeling choices made for "tractability" often have the effect of ruling out an interesting role for higher order beliefs.

We will discuss one example illustrating how higher order beliefs about fundamentals determine outcomes in a version of Diamond and Dybvig's (1983) model of intermediation and bank runs. In the environment described, there is a unique equilibrium. Thus for each possible "state of the world", we can determine whether there is a run, or not. But the "state of the world" is not determined only by the "fundamentals," i.e., the amount of money in the bank. Nor is the state determined by "sunspots," i.e., some payoff irrelevant variable that has nothing to do with fundamentals. Rather, what matters is depositors' higher order beliefs: what they believe about fundamentals, what they believe others believe, and so on. Our example illustrates why game theory confirms the common intuition that such higher order beliefs matter and determine outcomes. After the example, we will review a few attempts to incorporate this type of argument in models of financial markets.

The Example. There are two depositors in a bank. Depositor i 's type is ξ_i ; if ξ_i is less than 1, then depositor i has liquidity needs that require him to withdraw money from the bank; if ξ_i is greater than or equal to 1, he has no liquidity needs and maximizes expected return. If a depositor withdraws his money from the bank, he obtains a guaranteed payoff of $r > 0$. If he keeps his money on deposit and the other depositor does likewise, he gets a payoff of R , where $r < R < 2r$. Finally, if he keeps his money in the bank and the other depositor withdraws, he gets a payoff of zero.

Notice that there are four states of "fundamentals": both have liquidity needs, depositor 1 only has liquidity needs, depositor 2 only has liquidity needs and neither has liquidity needs. If there was common knowledge of fundamentals, and at least one depositor had liquidity needs, the unique equilibrium has both

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depositors withdrawing. But if it were common knowledge that neither depositor has liquidity needs, they are playing a co-ordination game with the following payoffs:

	Remain	Withdraw
Remain	R, R	0, r
Withdraw	r, 0	r, r

With common knowledge that neither investor has liquidity needs, this game has two equilibria: both remain and both withdraw. We will be interested in a scenario where neither depositor has liquidity needs, both know that no one has liquidity needs, both know that both know this, and so on up to any large number of levels, but nonetheless it is not common knowledge that no one has liquidity needs. We will show that in this scenario, the unique equilibrium has both depositors withdrawing. Clearly, it is then higher order beliefs in addition to fundamentals that determine the outcome.

Here is the scenario. The depositors' types, ξ_1 and ξ_2 , are highly correlated; in particular suppose that a random variable ω is drawn from a smooth distribution on the non-negative numbers and each ξ_i is distributed uniformly on the interval $[\omega - \epsilon, \omega + \epsilon]$, for some small $\epsilon > 0$. Given this probability distribution over types, types differ not only in fundamentals, but also in beliefs about the other depositor's fundamentals, and so on. To see why, recall that a depositor has liquidity needs if ξ_i is less than 1. But when do both depositors *know* that both ξ_i are more than or equal to 1? Only if both ξ_i are more than $1 + 2\epsilon$ (since each player knows only that the other's signal is within 2ϵ of his own)? When do both depositors know that both know that both ξ_i are more than 1? Only if both ξ_i are greater than $1 + 4\epsilon$. To see this, suppose that $\epsilon = 0.1$ and depositor 1 receives the signal $\xi_1 = 1.3$. She can deduce that ω is within the range 1.2-1.4 and hence that depositor 2's signal is within the range 1.1-1.5. However, if depositor 2 received the signal $\xi_2 = 1.1$, then he attaches a positive probability to depositor 1 having ξ_1 smaller than 1. Only if depositor 1's signal is greater or equal to $1 + 4\epsilon = 1.4$ would this possibility be avoided. Iterating this argument implies the following

result. It can never be common knowledge that both players are free of liquidity needs.

What do these higher order beliefs imply? In fact, for small enough ϵ , the unique equilibrium of this game has both depositors always withdrawing, whatever signals they observe. Observe first that by assumption each depositor must withdraw if ξ_i is smaller than 1, i.e., if she or he has liquidity needs. But suppose depositor 1's strategy is to remain only if ξ_1 is greater than some k , for $k > 1$. Further, consider the case that depositor 2 observes signal, $\xi_2 = k$. For small ϵ , he would attach probability about $1/2$ to depositor 1 observing a lower signal, and therefore withdrawing. Therefore depositor 2 would have an expected payoff of about $1/2R$ for remaining and r for withdrawing. Since $r > 1/2R$ by assumption, he would have a strict best response to withdraw if he observed k . In fact, his unique best response is to withdraw if his signal is less than some cut-off point strictly larger than k . But this implies that each depositor must have a higher cutoff for remaining than the other. This is a contradiction. So the unique equilibrium has both depositors always withdrawing.

This argument may sound paradoxical. After all, we know that if there was common knowledge that payoffs were given by the above matrix (i.e., both ξ_i were above 1), then there would be an equilibrium where both depositors remained. The key feature of the incomplete information environment is that while there are only four states of fundamentals, there is a continuum of states corresponding to different higher order beliefs. In all of them, there is a lack of common knowledge that both depositors do not have liquidity needs. Given our particular assumptions on payoffs, this is enough to guarantee withdrawal.

We do not intend to imply by the above argument that depositors are able to reason to very high levels about the beliefs and knowledge of other depositors. The point is simply that some information structures fail to generate sufficient common knowledge to support co-ordination on risky outcomes. How much common knowledge is "sufficient" is documented in the game-theory literature: what is required is the existence of "almost public" events, i.e., events that everyone believes very likely whenever they are true (see Monderer and Samet (1989) and Morris, Rob and Shin (1995)). While participants in financial markets may be unable to reason to very high levels of beliefs and knowledge, they should be able to recognize the existence or non-existence of almost public events.

The above example is a version of one introduced by Carlsson and van Damme (1993). Earlier work by Halpern (1986) and Rubinstein (1989) developed the link between co-ordination and common knowledge. See Morris and Shin (1997) for a survey of these developments. Morris and Shin (1998) generalize the logic of the above example to a model with a continuum of investors deciding whether or not to attack a currency with a fixed peg. Higher order beliefs are a key determinant of investors' ability to co-ordinate their behavior, and thus a key factor in determining when currency attacks occur.

A number of other models have explored the role of higher order beliefs in finance. In Abel and Mailath (1994), risk-neutral investors subscribe to securities paid from a new project's revenues. They note that it is possible that all investors subscribe to the new securities even though all investors' expected return is negative. This could not happen if it was common knowledge that all investors' expected returns are negative.

Allen, Morris and Postlewaite (1993) consider a rational expectations equilibrium of a dynamic asset trading economy with a finite horizon, asymmetric information and short sales constraints. They note that an asset may trade at a positive price, even though every trader knows that the asset is worthless. Even though each trader knows that the asset is worthless, he attaches positive probability to some other trader assigning positive expected value to the asset in some future contingency. It is worth holding the asset for that reason. Again, this could not occur if it were common knowledge that the asset was worthless.

Kraus and Smith (1989) describe a model where the arrival of information about others' information (not new information about fundamentals) drives the market. Kraus and Smith (1998) consider a model where multiple self-fulfilling equilibria arise because of uncertainty about other investors' beliefs. They term this "endogenous sunspots". They show that such sunspots can produce "pseudo-bubbles" where asset prices are higher than in the equilibrium with common knowledge.

Shin (1996) compares the performance of decentralized markets with dealership markets. While both perform the same in a complete information environment, he notes that the decentralized market performs worse in the presence of higher order uncertainty about endowments. The intuition is that

a decentralized market requires co-ordination that is sensitive to a lack of common knowledge, whereas the dealership requires less co-ordination.

Information Cascades

There is an extensive literature concerned with informational cascades. Welch (1992) is an early example. A group of potential investors must decide whether to invest in an initial public offering (IPO) sequentially. Each investor has some private information about the IPO. Suppose that the first few investors happen to observe bad signals and choose not to invest. Later investors, even if they observed good signals, would ignore their own private information and not invest on the basis of the (public) information implicit in others' decisions not to invest. But now even if the majority of late moving investors has good information, their good information is never revealed to the market. Thus inefficiencies arise in the aggregation of private information because the investors' actions provide only a coarse signal of their private information. This type of phenomenon has been analyzed more generally by Banerjee (1992) and Bikhchandani, Hirshleifer and Welch (1992). Finance applications are surveyed in Devenow and Welch (1996).

It is important to note that informational cascades occur even in the absence of any payoff interaction between decision makers. In the Welch (1992) account of initial public offerings, investors do not care whether others invest or not; they merely care about the information implicit in others' decisions whether to invest. But the argument does rely on decisions being made sequentially and publicly. Thus an informational cascades account of bank runs would go as follows. Either the bank is going to collapse or it will not, *independent of the actions of depositors*. Depositors decide whether to withdraw sequentially. If the first few investors happened to have good news, the bank would survive; if they happened to have bad news, the bank would not survive. By contrast, in the previous section, we described a scenario where despite the fact that all investors knew for sure that there was no need for the bank to collapse, it had to collapse because of a lack of common knowledge that the bank was viable. That scenario arose only because of payoff interaction (each depositor's payoff depends on other depositors' actions, because they influence the probability of collapse); but it occurred even when all decisions were made simultaneously.

One major weakness of the informational cascade argument is that it relies on action sets being too coarse to reveal private information (see Lee (1993)).

There are some contexts where this assumption is natural: for example, investors' decisions whether to subscribe to initial public offerings at a fixed offer price (although even then the volume demanded might reveal information continuously). But once prices are endogenized, the (continuum) set of possible prices will tend to reveal information. Researchers have identified two natural reasons why informational cascades might nonetheless occur in markets with endogenous price formation. If investors face transaction costs, they may tend not to trade on the basis of small pieces of information (Lee (1998)). In this case, market crashes might occur when a large number of investors, who have observed bad news but not acted on it, observe a (small) public signal that pushes them into trading despite transaction costs. Avery and Zemsky (1998) exploit the fact that although prices may provide rich signals about private information, if private information is rich enough (and, in particular, multi-dimensional), the market will not be able to infer private information from prices.

Heterogeneous Prior Beliefs

Each of the two previous topics we reviewed concerned richer models of asymmetric information. We conclude by discussing the more basic question of how differences in beliefs are modeled. A conventional modeling assumption in economics and finance is the common prior assumption: rational agents may observe different signals (i.e., there may be asymmetric information) but it is assumed that their posterior beliefs could have been derived by updating a common prior belief on some state space. Put differently, it is assumed that all differences in beliefs are the result of differences in information, not differences in prior beliefs.

For some purposes, it does not matter if differences in beliefs are explained by different information or differences in priors. For example, Lintner (1969) derived a CAPM with heterogeneous beliefs and – assuming, as he did, that investors do not learn from prices – the origin of their differences in beliefs did not matter. It is only once it is assumed that individuals learn from others' actions (or prices that depend on others' actions) that the difference becomes important. Thus the distinction began to be emphasized in finance exactly when game theoretic and information theoretic issues were introduced. Most importantly, “no trade” theorems, such as that of Milgrom and Stokey (1982), established that differences in beliefs based on differences in information alone could not lead to trade.

But while the distinction is undoubtedly crucial, this does not justify a claim that heterogeneous prior beliefs are inconsistent with rationality. See Morris (1995) for a review of attempts to justify this claim and also Gul (1998) and Aumann (1998). In any case, there is undoubtedly a significant middle ground between the extreme assumptions that (1) participants in financial markets are irrational; and (2) all differences in beliefs are explained by differences in information. We will briefly review some work in finance within this middle ground.

Harrison and Kreps (1978) considered a dynamic model where traders were risk neutral, had heterogeneous prior beliefs (*not* explained by differences in information) about the dividend process of a risky asset, and were short sales constrained in that asset. They observed that the price of an asset would typically be more than any trader's fundamental value of the asset (the discounted expected dividend) because of the option value of being able to sell the asset to some other trader with a higher valuation in the future. Morris (1996) examined a version of the Harrison and Kreps model where although traders start out with heterogeneous prior beliefs, they are able to learn the true dividend process through time; a re-sale premium nonetheless arises, one that reflects the divergence of opinion before learning has occurred. Thus this model provides a formalization of E. Miller's (1977) explanation of the opening market overvaluation of initial public offerings: lack of learning opportunities implies greater heterogeneity of beliefs implies higher prices.

The above results concerned competitive models and were, therefore, non-strategic. But heterogeneous prior beliefs play a similar role in strategic models of trading volume. Trading volume has remained a basic puzzle in the finance literature. It is hard to justify the absolute volume of trade using standard models where trade is generated by optimal diversification with common prior beliefs. Empirically relevant models thus resort to modeling shortcuts, such as the existence of noise traders. But ultimately the sources of speculative trades must be modeled and differences of opinion (heterogeneous prior beliefs) are surely an important source of trade.

In Harris and Raviv (1993), traders disagree about the likelihood of alternative public signals conditional on payoff relevant events. They present a simple model incorporating this feature that naturally explains the positive autocorrelation of trading volume and the correlation between absolute price changes and volume as well as a number of other features of financial market

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data. A number of other authors, Varian (1989) and Biais and Bossaerts (1998), have derived similar results. The intuition for these findings is similar to that of noise trader models. In our view, however, explicitly modeling the rational differences in beliefs leading to trade will ultimately deepen our understanding of financial markets.

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